2.7 Piecewise Functions

**Goals**
- Represent piecewise functions.
- Use piecewise functions to model real-life quantities.

**Your Notes**

**VOCABULARY**

Piecewise function  A function represented by a combination of equations, each corresponding to a part of the domain

Step function  A piecewise function whose graph resembles a set of stair steps

---

**Example 1  Evaluating a Piecewise Function**

Evaluate \( f(x) \) when (a) \( x = -3 \) and (b) \( x = -1 \).

\[
f(x) = \begin{cases} 
  x - 4, & \text{if } x < -1 \\
  3x - 1, & \text{if } x \geq -1 
\end{cases}
\]

a. \( f(x) = x - 4 \)

Because \( -3 < -1 \), use the first equation.

\[
f(-3) = -3 - 4 = -7
\]

b. \( f(x) = 3x - 1 \)

Because \( -1 \geq -1 \), use the second equation.

\[
f(-1) = 3(-1) - 1 = -4
\]

---

**Example 2  Graphing a Piecewise Function**

Graph this function: \( f(x) = \begin{cases} 
  -x + 1, & \text{if } x < 0 \\
  2x + 1, & \text{if } x \geq 0 
\end{cases} \)

**Solution**

To the left of \( x = 0 \), the graph is given by \( y = -x + 1 \).

To the right of and including \( x = 0 \), the graph is given by \( y = 2x + 1 \).

The graph is composed of two rays with common initial point \((0, 1)\).
Example 3  **Graphing a Step Function**

Graph this function: \( f(x) = \begin{cases} 
-1, & \text{if } -3 \leq x < -1 \\
1, & \text{if } -1 \leq x < 1 \\
3, & \text{if } 1 \leq x < 3
\end{cases} \)

**Solution**

The graph is composed of three line segments. The first line segment is given by the equation \( y = -1 \) and represents the graph when \( x \) is greater than or equal to \(-3\) and less than \(-1\).

The **solid** dot at \((-1, 1)\) indicates that \( f(-1) = 1 \).

The **open** dot at \((1, 1)\) indicates that \( f(1) \neq 1 \).

---

Example 4  **Writing a Piecewise Function**

Write equations for the piecewise function whose graph is shown.

**Solution**

To the **left** of \( x = 1 \), the graph is part of the line passing through \((-2, -2)\) and \((1, -1)\). An equation of this line is

\[ y = \frac{1}{3}x - \frac{4}{3} \].

To the **right** of and including \( x = 1 \), the graph is part of the line passing through \((1, 1)\) and \((3, 3)\). An equation of this line is given by \( y = x \). The equations for the piecewise function are:

\[ f(x) = \begin{cases} 
\frac{1}{3}x - \frac{4}{3}, & \text{if } x < 1 \\
x, & \text{if } x \geq 1
\end{cases} \]
Example 5 Using a Step Function

Parking costs $1 per hour or a $4 maximum for 6 hours. Write and graph a piecewise function for the costs.

Solution

For times up to one hour, the charge is $1. For each additional hour, the charge is an additional $1 until you reach $4. Let $t$ be the number of hours you park.

\[ f(t) = \begin{cases} 
1, & \text{if } 0 < t \leq 1 \vphantom{\frac{1}{2}} \\
\frac{2}{3}, & \text{if } 1 < t \leq 2 \\
\frac{3}{4}, & \text{if } 2 < t \leq 3 \\
\frac{4}{5}, & \text{if } 3 < t \leq 6 
\end{cases} \]

Checkpoint Complete the following exercises.

1. Graph $f(x)$ and evaluate for $x = 0$, $x = 2$, and $x = 3$.

\[ f(x) = \begin{cases} 
x - 2, & \text{if } x \leq 2 \\
-x - 2, & \text{if } x > 2 
\end{cases} \]

$-2, 0, -5$

2. Write an equation for the graph shown.

\[ f(x) = \begin{cases} 
-x - 2, & \text{if } x \leq -1 \\
\frac{1}{3}x + \frac{7}{3}, & \text{if } x > -1 
\end{cases} \]

3. Shipping costs $3 on purchases up to $10, $5 on purchases up to $50, and $8 on purchases over $50 up to $100. Write a piecewise function for this situation.

\[ f(x) = \begin{cases} 
3, & \text{if } 0 < x \leq 10 \\
5, & \text{if } 10 < x \leq 50 \\
8, & \text{if } 50 < x \leq 100 
\end{cases} \]